Lecture 2. Reconstruction Attacks.

- De-identified data ✗ ; releasing "Aggregate" Statistics?
- Warmup : Difference Attacks
- Reconstruction examples
- Reconstruction Formulation
  Linear Attacks [Dinur & Nissim 03]

What should "privacy" mean?

Warmup : Difference Attacks

Q: How many people were born on 1992 and live in Zipcode 15206 and have a heart disease?

A: 1 5 ≥

Q: How many faculty members @ CMU joined before 9/1/2020 and have had a heart disease?

A: 37

Q: How many faculty members @ CMU joined before 9/2/2020 and have had a heart disease?

A: 38

Facebook ad campaign targeting interface.

Ref: Korolova,
"Privacy violation Using
Microtargeted Ads: A Case Study"

Reconstruction in the US Census.

- 3 Males
- Ages $A \le B \le C$
- $1 \le A \le B \le C \le 125$
- Median = 30 $\Rightarrow B = 30$

  $A \le 30$, $C \ge 30$
- Mean $\frac{A+B+C}{3} = 44$

  $\Rightarrow A + C = 102$

(A, C) has 30 possible choices.

Before: $(125)^3$ ~~cho~~ possible choices

TABLE 1: FICTIONAL STATISTICAL DATA FOR A FICTIONAL BLOCK

		AGE		
STATISTIC	GROUP	COUNT	MEDIAN	MEAN
1A	total population	7	30	38
2A	female	4	30	33.5
2B	male	3	30	44
2C	black or African American	4	31	48.5
2D	white	3	24	24
3A	single adults	(D)	(D)	(D)
3B	married adults	4	51	54
4A	black or African American female	3	36	36.7
4B	black or African American male	(D)	(D)	(D)
4C	white male	(D)	(D)	(D)
4D	white female	(D)	(D)	(D)
5A	persons under 5 years	(D)	(D)	(D)
5B	persons under 18 years	(D)	(D)	(D)
5C	persons 64 years or over	(D)	(D)	(D)

Note: Married persons must be 15 or over

gender

race

Garfinkel, Abowd, Martindale 2018.

TABLE 2: POSSIBLE AGES FOR A MEDIAN OF 30 AND MEAN OF 44

A	B	C	A	B	C	A	B	C
1	30	101	11	30	91	21	30	81
2	30	100	12	30	90	22	30	80
3	30	99	13	30	89	23	30	79
4	30	98	14	30	88	24	30	78
5	30	97	15	30	87	25	30	77
6	30	96	16	30	86	26	30	76
7	30	95	17	30	85	27	30	75
8	30	94	18	30	84	28	30	74
9	30	93	19	30	83	29	30	73
10	30	92	20	30	82	30	30	72

Reconstruction in the US Census. 2010.

Variable	Range
Block	6,207,027 inhabited blocks
Sex	2 (Female/Male)
Age	103 (0-99 single age year categories, 100-104, 105-109, 110+)
Race	63 allowable race combinations
Ethnicity	2 (Hispanic/Not)
Relationship	17 values

Publication	Released counts
PL94-171 Redistricting	2,771,998,263
Balance of Summary File 1	2,806,899,669
Total Statistics in PL94-171 and Balance of SF1:	5,578,897,932
Published Statistics/person	18
Recall: Collected variables/person:	6
Published Statistics/collected variable	**18 ÷ 6 ffi 3**

5.5 billion simultaneous equations

on 1.8 billion unknown integers

Reconstruction Formulation

Dataset X

Statistics $f_1, \ldots, f_k$

answers

$$\begin{cases} a_1 \approx f_1(X) \\ a_2 \approx f_2(X) \\ \vdots \\ a_k \approx f_k(X) \end{cases} \longrightarrow \text{Constraints.}$$

Reconstruction Problem: Given "constraints" $\{f_i(X) \approx a_i\}$, find a dataset $\tilde{X}$ that is consistent w/ the constants.

Satisfiability Problem. ↓ Worse-case NP-hard.

look almost the same as X.

Linear Reconstruction Attack

- Introduced by Dinur & Nissim in 2003 ⟼ development of Differential Privacy. 06

Dataset →

Name	Postal Code	Age	Sex	Has Disease?
Alice	02445	36	F	1
Bob	02446	18	M	0
Charlie	02118	66	M	1
⋮	⋮	⋮	⋮	⋮
Zora	02120	40	F	1

$\mathcal{Z}$: identifiers — Secret bit

← Abstractly

Identifiers	Secret
z_1	s_1
z_2	s_2
z_3	s_3
⋮	⋮
z_n	s_n

Release count statistics: # people satisfy some property

- How many people are older than 40 & have secret bit = 1?

$\varphi(z_j)$

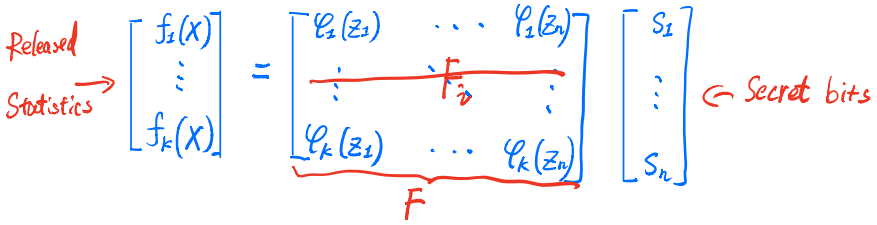
$$f(X) = \sum_{j=1}^{n} \varphi(z_j)\, s_j \quad \text{for some } \varphi : \mathcal{Z} \mapsto \{0,1\}$$

property on z_i

$$f(X) = \left(\varphi(z_1), \varphi(z_2), \ldots, \varphi(z_n)\right) \cdot \left(s_1, \ldots, s_n\right)$$

bit vector — Secret bits

Releasing k linear Statistics



$$f_i(X) = F_i \cdot s$$

Examples:

$\varphi_1(z_j) = 1$: z_j is older than 40

$\varphi_2(z_j) = 1$: z_j is older than 40 and male

$\varphi_3(z_j) = 1$: z_j is older than 20 and male

First Reconstruction Attack

"You can't release all count statistics with non-trivial accuracy."
if "privacy-preserving"

Queries: $k = 2^n$

For every $v \in \{0,1\}^n$, $F_v = v$

Reconstruction:

Suppose the answers $(a_v)_{v \in \{0,1\}^n}$, $\forall v \in \{0,1\}^n$, $|F_v \cdot s - a_v| \leq \alpha n$

(True answer, Released answer)

Choose $\tilde{s} \in \{0,1\}^n$, $\forall v$, $|F_v \cdot \tilde{s} - a_v| \leq \alpha \cdot n$

Constraints.

Theorem. $\|s - \tilde{s}\|_1 \leq 4\alpha n$

$\alpha = 5\%$

Reconstruct 80% of secrets.

Theorem. If all 2^n counts are within αn error, then $s, \tilde{s}$ disagree on $\leq 4\alpha n$ bits.

Proof Intuition.

a_f: Released answer

$s = [1011 \_\_\_\_\_]$ $f \cdot s$: true answer

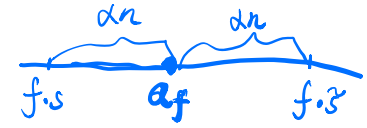
$\tilde{s} = [0100 \_\_\_\_\_]$ $\rightarrow |a_f - f \cdot s| \leq \alpha n$

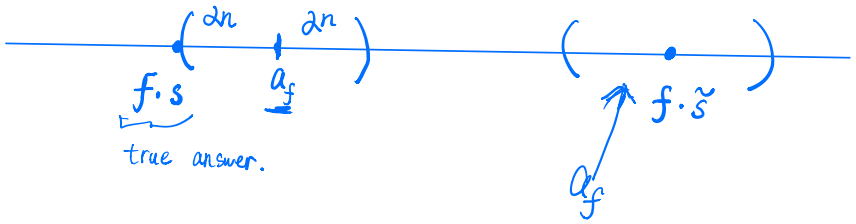
property that capture diff.

Reconstruction: find $\tilde{s}$

$\rightarrow |a_f - f \cdot \tilde{s}| \leq \alpha n$

$f \cdot \tilde{s} \approx a_f$





Theorem. If all 2^n counts are within αn error, then $s, \tilde{s}$ disagree on $\leq 4\alpha n$ bits.

Proof Sketch.

Two sets: $S_{01} = \{j : s_j = 0 \ \& \ \tilde{s}_j = 1\}$

$S_{10} = \{j : s_j = 1 \ \& \ \tilde{s}_j = 0\}$

If $\|s - \tilde{s}\|_1 > 4\alpha n$ ← Proof by contradiction.

$\Rightarrow |S_{01}| > 2\alpha n$ or $|S_{10}| > 2\alpha n$

$\Rightarrow$ there exists $v \in \{0,1\}$ such that $|v \cdot (\tilde{s} - s)| > 2\alpha n$ ← Statistic

$\Rightarrow |v \cdot \tilde{s} - a_v| > 2\alpha n - |v \cdot s - a_v| > \alpha n$

Triangle Ineq $\leq \alpha n$

$\Rightarrow$ Contradiction ← Contradiction

Reconstruction Using Fewer Queries

Released Statistics $<< 2^n$?

Dinur & Nissim

Attack : Choose $k = 20n$ random $\varphi_i : Z \mapsto \{0,1\}$, $\forall i \in [k]$.

$\Rightarrow$ k random vectors/queries $F_i \in \{0,1\}^n$

Suppose that answers : $\forall i \in [k]$, $|F_i \cdot s - a_i| \leq \alpha n$

Find $\tilde{s} \in \{0,1\}^n$ such that: $\forall i \in [k]$, $|F_i \cdot \tilde{s} - a_i| \leq \alpha n$

Theorem. $\|s - \tilde{s}\|_1 \leq 256\,\alpha^2 n^2$. w.h.p.

$\alpha \lesssim \frac{1}{\sqrt{n}}$